## ELEC50001 EE2 Circuits and Systems

## Problem Sheet 2 Solutions

## (Operation Amplifier Applications - Lectures 3-4)

1. The circuit here is also covered in Year 1 ADC Lecture 11. Q 1 and Q 2 needs 0.7 V in the baseemitter to make them conduct and act as emitter following for Load. So Q1a circuit waveform will look like this. (Although I discourage anyone to just remember names, this output circuit architecture is known as a Class B output.)


Adding Q1 and Q2 provides the necessary bias voltages $\mathrm{V}_{\mathrm{BE}}$ for Q 3 and Q 4 in the circuit in Figure Q1b. R3 and R4 are small (typically 10 ohms) and they improve the linearity of the amplifier. The waveform at Vout will look something like this. There is not cross-over distortion. This is also known as a class AB output.

2. We first need to determine what determines the biasing of this transistor and hence the quiescent base current. $\mathrm{V}_{\mathrm{BE}}$ is 0.7 V , therefore $\mathrm{R}_{\mathrm{B}}$ determines $\mathrm{I}_{\mathrm{BQ}}$.

$$
I_{B Q}=\frac{V c c-0.7}{R_{B}}=19.3 \mathrm{~mA}
$$

Therefore $I_{C Q}=\beta I_{B}=25 * 19.3 \mathrm{~mA}=0.48 \mathrm{~A}$

$$
V_{C E_{Q}}=V_{C C}-I_{C} R_{C}=20-0.48 * 20=10.4 \mathrm{~V}
$$

Note that the supply current is a sinusoidal wave centred around $I_{C Q}$.

Therefore supply input power is:

$$
P_{i}(d c)=V_{C C} I_{C_{Q}}=20 * 0.48=9.6 \mathrm{~W}
$$

Given that input (ac) signal has a peak current of 10 mA in the question, we can calculate the peak collector current:

$$
I_{C}(p k)=\beta I_{B}(p k)=250 m A(p k)
$$

Therefore ac power to load ( Rc ) is:

$$
P_{o}(a c)=I_{C}^{2}(r m s) R_{C}=\frac{I_{C}^{2}(p k)}{2} R_{C}=\frac{0.25^{2}}{2} * 20=0.625 \mathrm{~W}
$$

Hence the amplifier's power efficiency is only:

$$
\eta=\frac{P_{o}(a c)}{P_{i}(d c)}=\frac{0.625}{9.6} * 100 \%=6.5 \%
$$



Figure Q2
3. We solve this problem in a similar way to that of $Q 2$ except that the input power is calculated with current that is NOT the same as the quiescent current due to the push-pull action of Q1 and Q2. Instead we have to calculate the average current for the full-wave rectified current as explained in the notes.

The peak input signal voltage is:

$$
V_{i}(p k)=\sqrt{2} V_{i}(r m s)=\sqrt{2} * 12=17 V
$$

Assuming that the voltage gain is 1 . Therefore $V_{L}(p k)=17 \mathrm{~V}$
Hence $P_{o}(a c)=\frac{V_{L}^{2}(p k)}{2 R_{L}}=\frac{17^{2}}{2 \times 4}=36.125 \mathrm{~W}$
The peak load current is:

$$
I_{L}(p k)=\frac{V_{L}(p k)}{R_{L}}=\frac{17}{4}=4.25 \mathrm{~A}
$$

Therefore the average (dc) current draw from supply rails is:

$$
I_{d c}=\frac{2}{\pi} I_{L}(p k)=2 \times \frac{4.25}{\pi}=2.71 A
$$

Hence power from supply is

$$
P_{i}(d c)=V_{C C} I_{d c}=25 \times 2.71=67.75 W
$$

The power dissipated by each of the output transistors Q1 and Q2 is the same. It is:

$$
P_{Q}=\frac{P_{i}-P_{o}}{2}=\frac{67.75-36.125}{2}=15.8 \mathrm{~W}
$$

The amplifier efficiency is $\eta=\frac{P_{o}}{P_{i}} \times 100 \%=\frac{36.125}{67.75} \times 100 \%=53.3 \%$
This is much better than the efficiency found in Q2!


Figure Q2
4. $Z=[R 2 /(R 1+R 2)] * Y=X^{\prime} / K \quad$ (due to the negative feedback)

Therefore $\mathrm{X}^{\prime}=[\mathrm{R} 2 /(\mathrm{R} 1+\mathrm{R} 2)] * K * Y$
$Y=A_{1}\left(X-X^{\prime}\right)$
Therefore $Y=A_{1} X-A_{1} * K *[R 2 /(R 1+R 2)] * Y$

$$
\begin{aligned}
& \mathrm{Y}\left\{1+\mathrm{A}_{1} \mathrm{~K} *[\mathrm{R} 2 /(\mathrm{R} 1+\mathrm{R} 2)]\right\}=\mathrm{A}_{1} \mathrm{X} \\
& \text { Closed-loop gain }=\frac{Y}{X}=\frac{1}{\left(\frac{1}{A_{1}}+\frac{K R 2}{R 1+R 2}\right)}
\end{aligned}
$$

Check: Assume A1 = infinite, $K=1$, Gain = 1+ R1/R2 - correct for conventional non-inverting amplifier.


Figure Q4
5. Circuit for Q5a: Comparator output switch over state when $\mathrm{V}+$ reaches $\mathrm{V}_{\text {REF }}$. Apply $K C L$ at $\mathrm{V}+$ node: $\left(\mathrm{V}_{\text {IN }}-\mathrm{V}_{\text {REF }}\right) / R 1=\left(\mathrm{V}_{\text {REF }}-\mathrm{V}_{\text {OUT }}\right) / R 2=>\mathrm{V}_{\text {IN }}=(1+R 1 / R 2) \mathrm{V}_{\text {REF }}-(R 1 / R 2)^{*} \mathrm{~V}_{\text {OUT }}$

For positive going $\mathrm{V}_{\text {IN }}, \mathrm{V}_{\text {OUT }}=0$. Therefore $\mathrm{V}_{\text {th_H }}=\mathrm{V}_{\text {REF }}(1+\mathrm{R} 1 / R 2)$.
For negative going $V_{I N}, V_{\text {OUT }}=5$. Therefore $V_{\text {th_L }}=V_{\text {REF }}(1+R 1 / R 2)-5 * R 1 / R 2$

Circuit for $Q 5 b$ : Apply $K C L$ at node $V+: V_{I N}=V_{R E F}[R 2 /(R 1+R 2)]+V_{\text {Out }}[R 1 /(R 1+R 2)]$.
For positive going $V_{I N}, V_{\text {OUT }}=5$. Therefore $V_{\text {th_H }}=V_{\text {REF }}[R 2 /(R 1+R 2)]+5 *[R 1 /(R 1+R 2))$.
For negative going $V_{I N}, V_{\text {OUT }}=0$. Therefore $\left.V_{\text {th_L }}=V_{\text {REF }} R 2 /(R 1+R 2)\right]$


Figure Q5a


Figure Q5b
6. Since $\mathrm{R} 1=180 \mathrm{k}$ and $\mathrm{R} 2=200 \mathrm{k}$, from $\mathrm{Q} 5, \mathrm{~V}_{\text {th_H }}=4.75 \mathrm{~V}$ and $\mathrm{V}_{\text {th_L }}=0.25 \mathrm{~V}$. Therefore the triangular signal peak-to-peak is 4.5 V .

For the integrator, the integration current $\mathrm{Ic}= \pm(\mathrm{Vsq}-2.5) / \mathrm{R}, \quad(\mathrm{R}=10 \mathrm{k})$.
Therefore $\Delta \mathrm{Vc} / \Delta \mathrm{t}=\mathrm{ic} / \mathrm{C}=2.5 / \mathrm{RC} . \Delta \mathrm{Vc}=4.5 \mathrm{~V}$. Therefore $\Delta \mathrm{t}=\mathrm{RC} 4.5 / 2.5=18 \mathrm{uS}$. Therefore the oscillation frequency is $1 /\left(2^{*} 18\right) \mathrm{MHz}=27.8 \mathrm{kHz}$..


Figure Q6
7. $\frac{\operatorname{Vout}(s)}{\operatorname{Vin}(s)}=\frac{1 / R 1 R 2 C 1 C 2}{s^{2}+s\left(\frac{1}{R 2 C 1}+\frac{1}{R 1 C 1}\right)+\frac{1}{R 1 C 1 R 2 C 2}}$

For Butterworth filter, the cut-off frequency is $f c=\frac{1}{2 \pi \sqrt{R 1 R 2 C 1 C 2}}$
Since $\mathrm{C} 1=\mathrm{C} 2=10 \mathrm{nF}, \mathrm{R} 1=\mathrm{R} 2=1.59 \mathrm{k}$ ohm.


Figure Q7
8. The triangular signal from Q 5 goes from 0.25 V to 4.75 V . The range is 4.5 V . Therefore:

Vpwm(average) $=(\operatorname{Vin}-0.25) *(5 / 4.5)$ for $0.25 \leq \operatorname{Vin} \leq 4.75$.
Check: When Vin $=0.25 \mathrm{~V}$, Vpwm (average) $=0 \mathrm{~V}$ (or $0 \%$ duty cycle). When Vin $=4.75 \mathrm{~V}$, Vpwm(average) $=5 \mathrm{~V}$ (or $100 \%$ duty cycle).


Figure Q8

